BT-1034.03

Stresses and Configurations of Natural-Shaped Balloons

Litton Systems, Inc.

Justin H. Smalley

Dec 1963

BT- 1034.03

III Stresses and Configurations of Natural-Shaped Balloons

Justin H. Smalley
Applied Science Division
Litton Systems, Inc.

Abstract

The equations derived in the early work by the University of Minnesota for the stresses in free balloons have been verified. The coordinates for natural-shaped balloons as determined by the University on an analog computer have been refined by digital computer techniques. In addition, a larger class of balloon shapes have been computed, including larger sigma values, both super- and sub-pressures, non-zero circumferential stresses, and other than "flat-top" balloons. The meridional stresses have also been investigated.

1. INTRODUCTION

This is a report on some of my investigations of the theoretical shapes and stresses of fully-inflated free balloons. The most recent published report on the shape of balloons appeared in 1953[1] and concerned work performed at the University of Minnesota. Since that time, new manufacturing techniques have been devised, new materials have become available, and new operational methods have been put into use Thus a new look at balloon design was indicated. Furthermore, I was interested in comparing my results, using a digital computer, with those of the University, which utilized an analog computer.

2. DISCUSSION

With these items in mind, I began a search of the literature for earlier papers on the shape of inflated devices. The earliest references that I found were to graphical[2] and mechanical[3] solutions for the shape of a lighter-than-air craft, such as an airship, this work was done in 1914. These authors assumed that the weight

(Author's manuscript received for publication 31 December 1963)

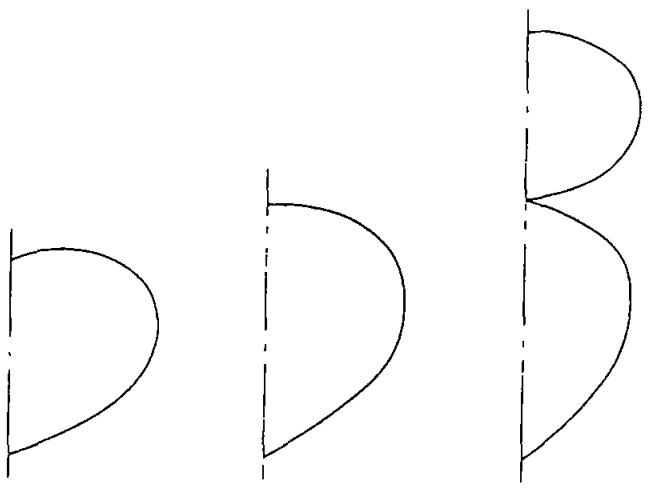
of the material was negligible. In Reference 3 a closed solution to this same problem, in terms of elliptic integrals, was reported. In the determination of the shape of an airship it is possible to separate the meridional and longitudinal stresses. With some modification, these same works would be applicable to the shape of a balloon with zero circumferential stress. It was also found that there is a considerable body of literature on parachutes where studies had been made of the shape of inflated objects of that kind. For their purposes, these investigators assumed that the pressure inside the parachute remained constant.

The next significant contribution was the development of the "natural shape" by the University of Minnesota, and the publication of their famous sigma tables.[1] These authors assumed that the material was thin, perfectly flexible, and non-extensible and that the solution would be a balloon which is rotationally symmetric about a vertical axis. Furthermore, they assumed that the density of the gas within the balloon and that of the surrounding air was constant and they also specified that the circumferential stress should be zero. I have used the same assumptions, except that I have, in addition, considered the case of non-zero circumferential stress.

In the development of the equations, it is convenient to non-dimensionalize the equations. In so doing, the expression for sigma falls out. Sigma is nothing more nor less than the unit weight of the balloon material made non-dimensional. Sigma is a very important parameter for two reasons If we make an assumption concerning the general shape of the balloon--for example, one having a flat top and we make an assumption concerning the circumferential stress (zero, for example)--then sigma uniquely determines the shape of the balloon. In addition, sigma is made up of three factors: the unit weight of the balloon material, the balloon payload, and the buoyancy of the gas at the float altitude. These three factors are those most apt to be known by the designer at the outset of his design. Thus, having made the general assumptions mentioned above, the designer can, with very little effort, determine the shape, volume, and size of his balloon. I do not mean to oversimplify this process but, in principle, this is what occurs.

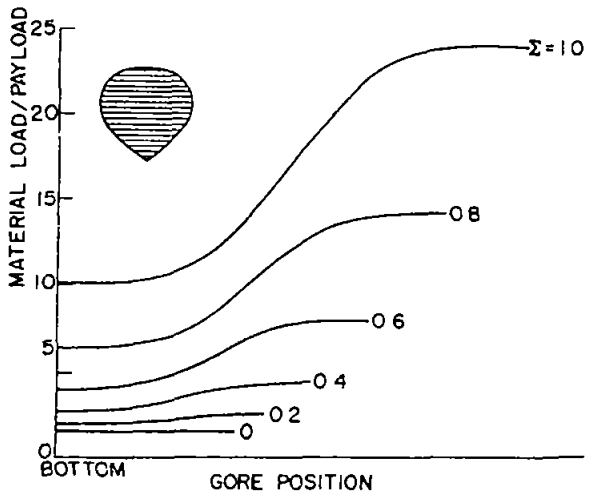
In my calculations, I used a modification of the Runge-Kutta method for the solution of simultaneous differential equations. I do not have a closed solution The solution is found by trial and error. Some of my results are indicated in the following figures.

Figure 1 shows several variations on a natural-shape balloon. I would like to see the term "natural-shape" reserved for balloons with zero circumferential stress. By zero super-pressure is meant that the pressure inside the balloon at the bottom apex is equal to the pressure of the outside atmosphere at that point. In Figure 1, the middle shape represents the standard flat-top balloon. Many different sizes with



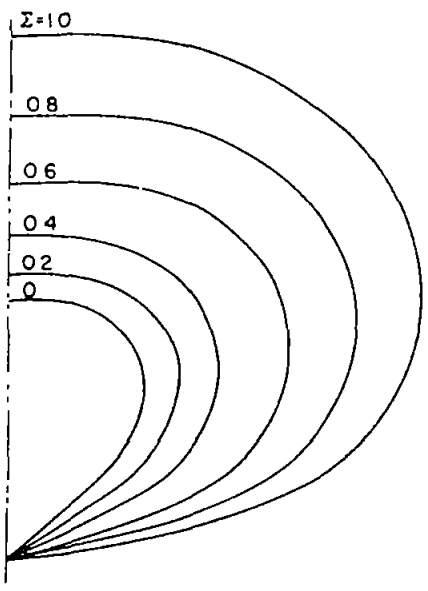
ZERO CIRCUMFERENTIAL STRESS
ZERO SUPERPRESSURE
Σ=02

Figure 1. Variations on a Natural-Shape Balloon



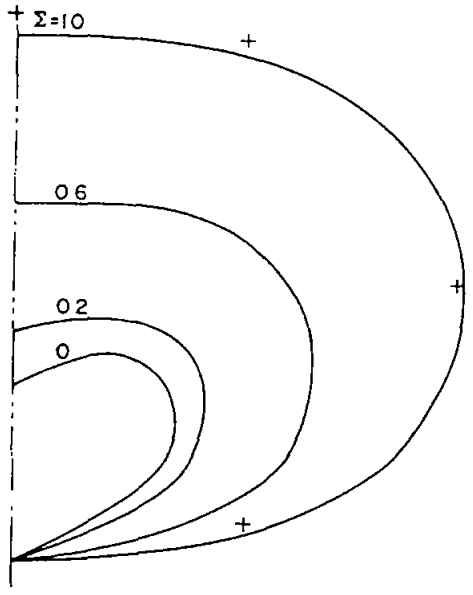
ZERO SUPERPRESSURE
ZERO CIRCUMFERENTIAL STRESS
FLAT TOP

Figure 3. Total Meridional Load in Balloon Material



ZERO SUPERPRESSURE
ZERO CIRCUMFERENTIAL STRESS

Figure 2. Balloon Shapes, Flat Top



ZERO SUPERPRESSURE
ZERO CIRCUMFERENTIAL STRESS

Figure 4. Balloon Shapes, One-Half Load at Top

many different weights and types of material have been flown. The shape on the left is a balloon with part of the payload mounted at the top. A variation on this design could involve a cord or cable connected from the top to the bottom of the balloon with all the payload mounted at the bottom. Another variation might involve several cords from the payload at the bottom to several points at the top. The shape at the right is a double-balloon shape. Although the total payload is carried at the bottom, part of the payload is supported by the upper balloon. An obvious variation of this design would be a change in the size of the upper balloon. There is no fundamental reason why there could be no more than two balloons. There could be three or even more. In any case, the top balloon will have a flat top.

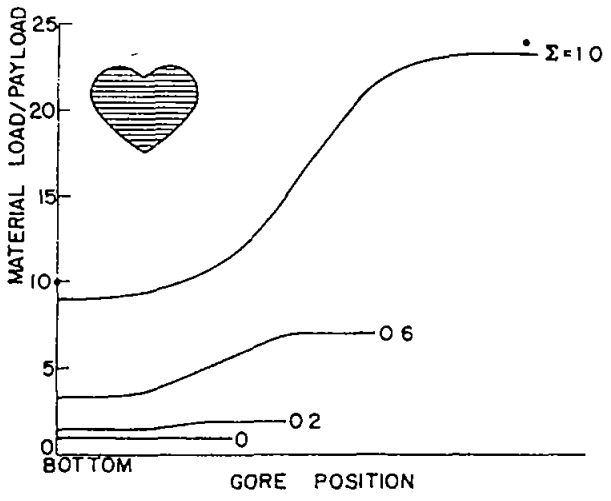
Figure 2 is the result of calculating the shape of the flat-top balloons when sigma ranges from zero to 1.0. This range will cover practically all balloon designs. The University of Minnesota calculated only shapes for sigma from zero to 0.4. It is of particular interest to note that all of these shapes will lift exactly the same payload. The smallest balloon (sigma equal to zero) has just enough volume to lift the payload. The larger volumes represented by the larger shapes are required to lift the payload plus the weight of their own balloon. The largest balloon has a volume approximately 22 times that of the smallest balloon.

Figure 3 shows the total load in the balloon material, that is, the vertical load tangent to the surface of the balloon. The same range of sigmas is used as given in Figure 2. For sigma equal to zero, the total film load is constant from the bottom to the top of the balloon. As sigma increases, the total film load increases until, for sigma equal to 1.0, the total film load at the top of the balloon is approximately 24 times the payload itself. For large sigma, the payload becomes a relatively small portion of the total weight of the system.

Figure 4 shows shapes of balloons that have some load at the top. For this particular design, half the load is mounted at the top. Again, each of these shapes will lift exactly the same payload. The crosses marked near the curve for sigma equal to 1.0, represent the shape of the balloon that has no load at the top.

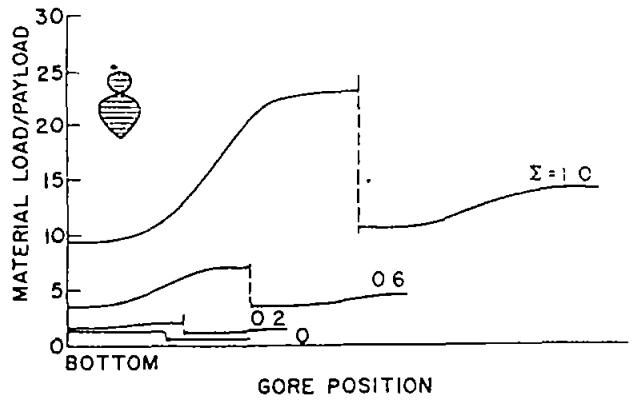
Figure 5 again shows the total load in the balloon material for balloons with half the load at the top. The shape of the curves is much the same as for the flat-top balloon. The dots indicate the position of the corresponding sigma-equal-to-1.0 curve for flat-top balloons. It is seen that the material load has been somewhat reduced when part of the load is at the top of the balloon. As a result, for the same payload one could use a slightly lighter material in this design of balloon.

Figure 6 shows three different shapes of double balloons. I have chosen to present balloons wherein half of the load is carried by the upper balloon. For comparison, the dotted curves are the corresponding shapes of the flat-top balloon for the same payload. I wish to call your attention to the upper balloons. In each case, the same sigma was used for both the upper and lower balloons. The two



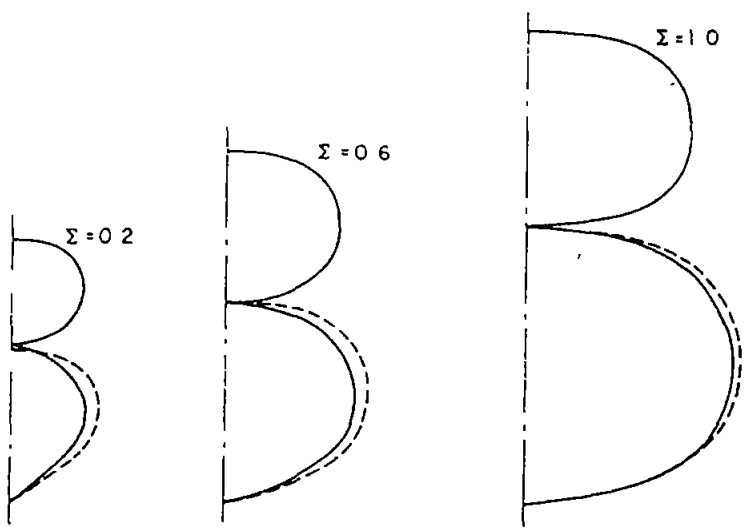
ZERO SUPERPRESSURE
ZERO CIRCUMFERENTIAL STRESS
ONE-HALF LOAD AT TOP

Figure 5. Total Meridional Load in Balloon Material, One-Half Load at Top



ZERO SUPERPRESSURE
ZERO CIRCUMFERENTIAL STRESS
DOUBLE BALLOON, ONE-HALF LOAD ON EACH

Figure 7. Total Meridional Load in Balloon Material, Double Balloon



ZERO SUPERPRESSURE
ZERO CIRCUMFERENTIAL STRESS
ONE-HALF OF LOAD SUPPORTED BY EACH BALLOON

Figure 6. Double-Balloon Shapes

would be much the same shape except that the upper balloon is subjected to a super-pressure. The super-pressure is represented by the height of helium in the lower balloon. Figure 7 presents the total material load in the double-balloon design. The striking difference in these curves is the reduction in film load in the upper balloon. These results are for the same sigma in both the lower and upper balloons, but with such a reduction in material one would use a considerably reduced sigma. I conclude that from purely load-carrying considerations, the fully inflated double-balloon system would be heavy compared to other designs.

Figure 8 summarizes some of the weight information for these natural-shape balloon designs. These data are for a sigma of 0.2. The bottom curve compares the weight-to-payload ratio for balloons with zero load at the top--increasing to an abscissa value of 1/2 where one half the payload is at the top of the balloon, and decreasing to an abscissa value of -1/2 where one half the payload is supported by a second balloon. Again, it is seen that the double-balloon system is heavy. The upper curve compares the weight-to-payload ratio for balloons with various values of super-pressure. One method for obtaining super-pressure would be to hang a duct below the bottom of the balloon. If the duct length were equal to the height of the balloon, the abscissa value would be 1. It is seen that balloons with negative values of the abscissa, that is, sub-pressure, are heavy. The minimum weight for a balloon occurs when the duct length is approximately one half the height of the balloon.

The foregoing have all had the circumferential stress specified as zero. When considering non-zero circumferential stress, I have found that the variety of balloons becomes so great that it is not practical to attempt to give generalized balloon designs. For that reason I have investigated a balloon which is designed to lift 40 pounds to 70,000 feet. One and one half mil Mylar material was used with a super-pressure of 5 millibars. This results in a sigma of 0.24 (Figure 9). These two curves are for a balloon with zero circumferential stress and one with a circumferential stress of 5 pounds per inch. Both of these balloons are wider than the corresponding spherical balloon. Figure 10 presents the stresses in the balloon. In this case, I have chosen to present actual stresses rather than total film loads. As one might expect, introduction of a circumferential stress results in a reduction in the corresponding meridional stress. The two balloons have constant circumferential stresses of zero and 5 pounds per inch, respectively.

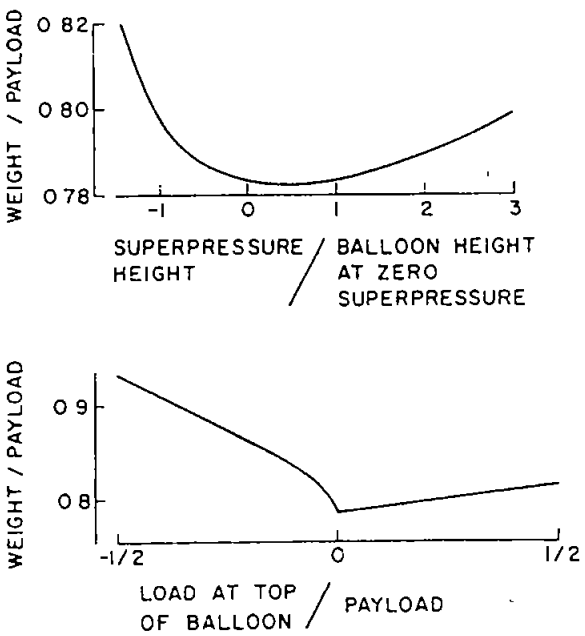


Figure 8 Weight of Balloons

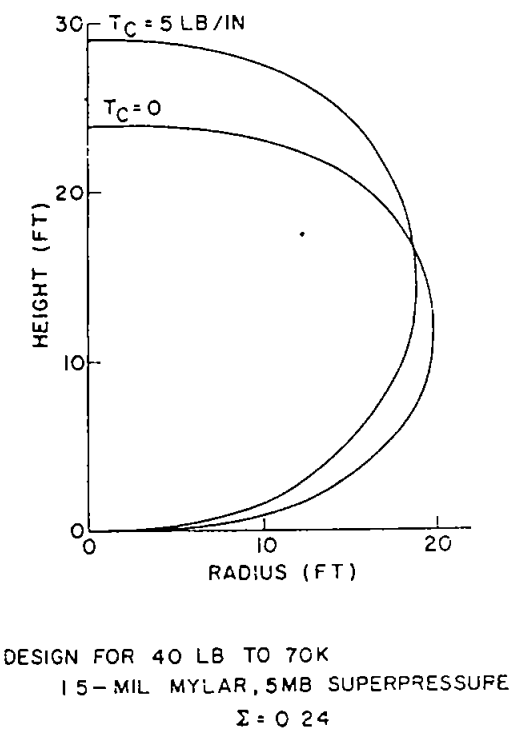


Figure 9. Effect of Circumferential Stress

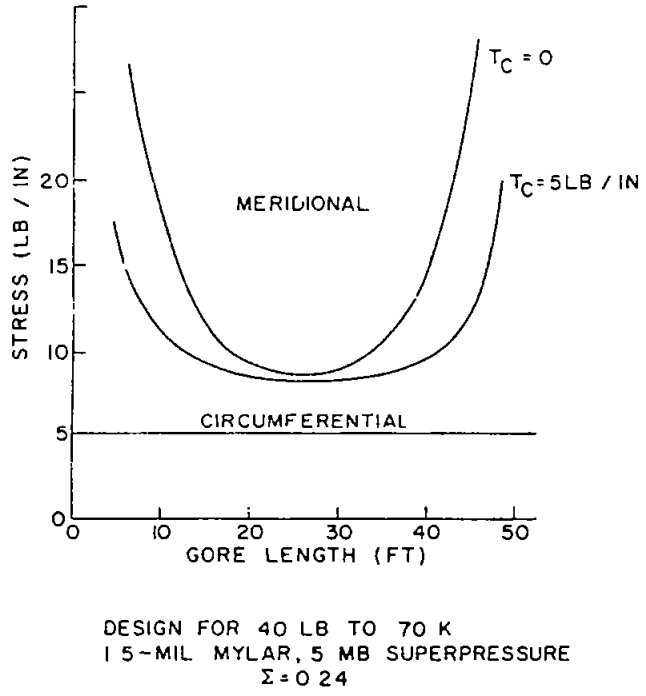


Figure 10. Comparison of Stresses

3. SUMMARY

In conclusion, the results of my investigation have been: The work of the University of Minnesota has been verified, although there was never any doubt as to the validity of their work. The sigma tables have been extended to cover most balloon designs. Various shapes have been studied and ways to reduce weight have been investigated, some methods of reducing their weight have been shown here. Certain knowledge of the stresses in the balloons has been determined. Finally, as a result of my work in calculating the shapes, I believe that special designs should be investigated carefully using computer analysis rather than by reliance on sigma tables alone.

References

1. Minnesota, University, Department of Physics. Research and development in the field of high altitude plastic balloons. Contract NOnr-710(01). Progress Report, Vol 9 (Dec. 22, 1952 - Dec. 3, 1953).

2. Predetermination de la forme d'un ballon gonfle. Technique Automobile et Aerienne 12, 112 11-14 (1921).

3 Blakemore, T L. and W.W. Pagon. Pressure airships, New York, Ronald Press, 1927